Key

Math 1 Name______________________

4-3 Exponential Growth Part 2 Date__________

- *I can define an exponential function.*
- *I can determine if a function is linear or exponential given the sequence, graph or table of values.*
- *I can identify the quantity being compared and write explicit/recursive equations to describe a real-world problem.*
- *I can use technology to find the point where two functions intersect.*
- *I can determine the practical domain and range in the context of a problem. And explain how they are related to the graph.*

1. Imagine that you found a lottery ticket in a parking lot that turned out to be the sole winning ticket for a $10,000 prize. Upon claiming your prize, you are offered two options:

   Option 1: Receive a single payment of $10,000 now.
   Option 2: Receive a single payment of $20,000 in ten years.

   a. Which option would you choose and why?

   $10,000 now because I'm impatient :-P

   b. Suppose a financial planner from your bank calls and offers another option.

   Option 3: Take the $10,000 now and invest all of the money into a certificate of deposit (CD) earning 8% interest compounded annually (annual means yearly) for ten years.

   Would you stick with the option chosen in part a, or switch to option 3? Why?

   Definitely take the $10,000 + invest it.

You hopefully are thinking, "I wonder how much money I would have in ten years using option 3."

Take notes below on how to calculate the balance in your account after each year from option 3.

1 year: $\underbrace{10{,}000 \cdot 0.08}_{\text{amount from interest}} + \underset{\text{Starting amount}}{10{,}000} = \$10{,}800$ Longer, icky method.

2 years: $10{,}800 \cdot 0.08 + 10{,}800 = \$11664$

Shorter, more awesome method

1 year: $10{,}000 \cdot \underbrace{1.08}_{108\%} = 10{,}800$

2 years: $10{,}800 \cdot 1.08 = 11664$

or $10{,}000 \cdot 1.08 \cdot 1.08 = 11664$

2. Write a recursive equation to model option 3.

$$\begin{cases} a_0 = 10{,}000 \\ a_n = a_{n-1} \cdot 1.08 \end{cases}$$

3. Write an equation in function notation that would model the amount of money in the account for any number of years x.

$$f(x) = 10{,}000\,(1.08)^x$$

4. Which equation is best to use to determine the amount of money in the account after 10 years, the recursive equation or the explicit function? Why?

Explicit! We don't have to find years 2-9 first. Just substitute 10 for x.

5. How much money will be in the account after ten years?

$$f(10) = 10{,}000\,(1.08)^{10} \approx \boxed{\$21{,}589.25}$$

6. How much more money would you get by using that plan as opposed to option 2 which paid $20,000 in ten years?

$21{,}589.25 - 20{,}000 = \$1{,}589.25$

7. What is the Average rate of change for the following years:

a. Years 1-2

$11{,}664 - 10{,}800$
(year 2) (year 1)
$= \boxed{\$864 \text{ per year}}$

b. Years 4-5

$14{,}693.28 - 13{,}604.89$
$= \boxed{\$1088.39 \text{ per year}}$

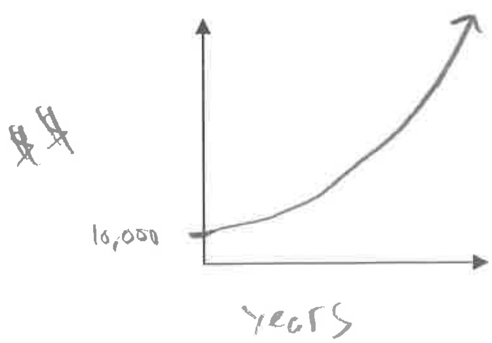
c. Years 9-10

$21589.25 - 19990.05$
$= \boxed{\$1{,}599.20 \text{ per year}}$

8. Is the average rate of change constant? If not, why?

No. This is not a linear pattern; it is exponential!

9. Draw a sketch of what the graph would look like and describe what the pattern.



Amount of money in the bank account increases at an increasing rate (slow at first the faster).

10. What domain and range would make sense for this situation?

Domain: Any number ≥ 0
$x \geq 0$

Range: Any numbers $\geq 10{,}000$ or $\{10000, 10000.01, 10000.02, \ldots\}$
$y > 10{,}000$

11. Using a graph's page in your calculator, find the solution to the following equations and explain what the solution tells you.

a. $10{,}000(1.08^x) = 50{,}000$ Solution: 20.91

Explanation: After 20.91 years, you will have \$50,000 in the account.

b. $10{,}000(1.08^x) \geq 25{,}000$ Solution: $x \geq 11.91$

Explanation: After 11.91 or more years, You will have at least \$25,000 in the bank.

c. $10{,}000(1.08^x) \leq 30{,}000$ Solution: $x \leq 14.27$

Explanation: You'll have \$30,000 or less if you leave the money in the bank for 14.27 or less years.

d. $10{,}000(1.08^x) = 10{,}000$ Solution: $x = 0$

Explanation: You have \$10,000 in the account from the beginning.

12. How much would be in an account after 10 years if \$15,000 were deposited at 4% annual interest compounded yearly?

$B(x) = 15000(1.04)^x$ $B(10) = 15000(1.04)^{10} \approx \boxed{\$22{,}203.66}$

13. How much would be in an account after 10 years if \$5,000 were deposited at 9.7% annual interest compounded yearly?

$B(x) = 5000(1.097)^x$ $B(10) = 5000(1.097)^{10} \approx \boxed{\$12{,}619.33}$

Compare the two savings plans from questions 12 and 13 and Option 3 from the previous problems.

14. Which one yielded the most money after 10 years?

Savings plan from #12.

15. Did any of the accounts double the initial amount of money deposited? If so, which one?

Yes, Option 3 + plan from #13.

16. Did any of the account triple the initial amount of money deposited? If so, which one?

No.